

# EVA 2011 - Abstracts of Contributed Posters

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## Extreme events cutoff, long-range correlation and linearisation effect in multifractal analysis

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### Abstract

The analysis of the linearization effect in multifractal analysis [2, 3], and hence of the estimation of moments for multifractal processes, is revisited borrowing concepts from the statistical physics of disordered systems, notably from the analysis of the so-called Random Energy Model [4]. Considering a standard multifractal process (compound Poisson motion [5]), chosen as a simple representative example, we show: i) the existence of a critical order  $q^*$  beyond which moments, though finite, cannot be estimated through empirical averages, irrespective of the sample size of the observation; ii) that multifractal exponents necessarily behave linearly in  $q$ , for  $q > q^*$ . Tayloring the analysis conducted for the Random Energy Model to that of Compound Poisson motion, we provide explicative and quantitative predictions for the values of  $q^*$  and for the slope controlling the linear behavior of the multifractal exponents. These quantities are shown to be related only to the definition of the multifractal process and not to depend on the sample size of the observation. Monte-Carlo simulations, conducted over a large number of large sample size realizations of compound Poisson motion, confirm and extend these analyses.

### References

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## Interval Estimation for the Parameters of the Gumbel Distribution

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### Abstract

The Gumbel distribution is one of the most popular widely used distributions in engineering. It is also known as the extreme value distribution of type I (Johnson et al., 1994). This distribution is frequently used for modeling a broad variety of extreme data from environmental, engineering and actuarial sciences. In this paper, we present exact confidence intervals and joint confidence region for the parameters of Gumbel distribution. A simulation study is performed to compare the proposed confidence intervals and joint confidence region. Two numerical examples with real data set and simulated data, are presented to illustrate the proposed methods. The results can be used for constructing exact confidence intervals and joint confidence region for the parameters of inverse Weibull distribution as well.

### References

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# Multivariate extreme values for assessing simultaneous over-exposure to food chemicals

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## Abstract

If eating is the privileged way of providing the necessary nutrients for the human organism, it also conveys toxic elements that, due to various environmental causes, contaminate the food. When consumed over certain tolerable doses, these can have a non-negligible impact on health. Similar phenomenons also occur when diets are either too rich or too poor in nutrients. For public health organisms such as the ANSES (the French agency for food, environmental and occupational health safety), it is then of major interest to understand the mechanisms that lead to over- or under-exposure to various chemical elements and quantify the level of risk in a given population. In the present study, we focus our attention on the relationships between high - and relatively rare - exposures to various nutrients and contaminants, that are assessed with the use of the multivariate extreme value theory.

Our variables of interest, i.e. exposures to a group of nutrients or contaminants, are not directly available; they result from a combination of both the consumption data base INCA2 (the national individual survey about eating habits in France) and the contamination data bases TDS2 (total dietary survey) and CIQUAL (information centre about food quality). Thus, extreme exposures can arise from over-consumption of products containing these elements, from ingestion of food with excessively high amounts of contaminants, or even from the association of moderately contaminated foodstuffs. To take all three situations into account, we estimate exposures with an incomplete generalized U-statistic as developed in P.Bertail and J.Tressou [1] and then study the dependence of the multivariate extremes with classical methods.

In reference to the previous work of J.Tressou and al. [2] on the probability of over-exposure to methylmercury, we focus on the case of heavy-tailed marginals. After a brief univariate analysis, we determine pairs of variables that can be studied simultaneously by assessing the coefficient of tail dependence and testing for asymptotic independence as in [3]. For seemingly dependent pairs we compare various estimates of the spectral measure, based on either parametrical or non-parametrical reductions to standard Fréchet.

From these results we draw conclusions about the probabilities of simultaneous over-exposure to methylmercury, dioxins and PCB, iron and sodium.

## References

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## Random field with random dimensions

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## Abstract

We study the limiting distribution of the maximum value of a stationary bivariate random field. In a first part, when the double dimensions have geometric growing pattern, a max-semistable distribution is obtained. Moreover, in a second part, considering the random field with geometric growing random dimensions, a mixture distribution is established for the maximum.

## References

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# Estimation of a new parameter discriminating between Weibull tail-distributions and heavy-tailed distributions

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## Abstract

The Gnedenko theorem is a general result in extreme value theory establishing the asymptotic distribution of extreme order statistics. The maximum of a sample of independent and identically distributed random variables after proper renormalization converges in distribution to one of the three possible maximum domain of attraction: Fréchet, Weibull and Gumbel. In a lot of applications (hydrology, finance, etc ...), the Fréchet maximum domain of attraction and the Gumbel maximum domain of attraction are used. The Gumbel maximum domain of attraction encompasses a large variety of distributions. Here, we focus on a subfamily of distributions called Weibull tail-distributions which depends on the Weibull tail-coefficient. Numerous works are dedicated to the estimation of this coefficient (see for instance, [1, 2, 3]), and to the estimation of the tail index (see [5] for a review). In order to explain why the same methodology can be used to estimate the Weibull tail-coefficient and the tail index, the authors proposed in [4] a family of distributions which encompasses the whole Fréchet maximum domain of attraction as well as Weibull tail-distributions. These distributions depend on 2 parameters  $\tau \in [0, 1]$  and  $\theta > 0$ . The first one,  $\tau$  allows us to represent a large panel of distribution tails ranging from Weibull-type tails ( $\tau = 0$ ) to distributions belonging to the maximum domain of attraction of Fréchet ( $\tau = 1$ ). The main goal of this communication is to propose an estimator for  $\tau$  independent of  $\theta$ . Under some assumptions we establish the asymptotic distribution of this estimator.

## References

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## On a Generalized Pickands estimator

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## Abstract

We consider the Pickands process

$$P_n(s) = \log(1/s)^{-1} \log \frac{X_{n-k+1,n} - X_{n-[k/s]+1,n}}{X_{n-[k/s]+1,n} - X_{n-[k/s^2]+1,n}},$$

$$\left( \frac{k}{n} \leq s^2 \leq 1 \right),$$

which is a generalization of the classical Pickands estimate  $P_n(1/2)$  of the extremal index. We undertake here a purely stochastic process view for the asymptotic theory of that process by using the Csörgő-Csörgő-Horvath-Mason (1986) [1] weighted approximation of the empirical and quantile processes to suitable Brownian bridges. This leads to the uniform convergence of the margins of this process to the extremal index and a complete theory of weak convergence of  $P_n$  in  $\ell^\infty([a, b])$  to some Gaussian process

$$\{\mathbb{G}, a \leq s \leq b\}$$

for all  $[a, b] \subset ]0, 1[$ . This frame greatly simplifies the former results and enable applications based on stochastic processes methods.

## References

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## DPOT Methodology: a duration based POT method with application to VaR

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### Abstract

Threshold methods, based on fitting a stochastic model to the excesses over a threshold, were developed under the acronym POT (peaks over threshold). In order to eliminate the tendency to clustering of violations, a model based approach within the POT framework, that uses the durations between excesses as covariates, is presented. This is named by DPOT methodology. Based on this approach, models for forecasting one-day-ahead Value-at-Risk were applied to real data. Comparative studies provide evidence that they can perform better than state-of-the art risk models and much better than the widely used RiskMetrics model.

### Acknowledgements

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## Modeling monthly maximum 24 hour precipitation in Iceland

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### Abstract

We develop a Bayesian hierarchical model for the prediction of monthly maximum 24 hour precipitation in Iceland. This model incorporates observed data, outputs from meteorological models and other covariates. The data are modeled with the generalized extreme value distribution. The hierarchical model is similar to that of [1]. Parametric inference is based on the Bayesian approach.

## References

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## Pricing in a consistent way both ground-up and excess of loss layers: an application to Liability Insurance loss data

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## Abstract

The usual practice of insurance companies, when modeling claim size distributions, is to fit classical models (often Lognormal and Pareto) to the data. However, when pricing high excess layer insurance contracts, it is critically important to obtain a good estimate for the right tail of the distribution. Extreme Value Theory (EVT) may be used for this purposes (see [1] and [2], among many others). On the other hand, in some cases it is desirable to obtain a model able to give a good estimation of insurance losses over the entire range of values observed in historical data, to price both ground-up and excess of loss layers in a consistent way.

In this paper we combine EVT and classical models, in order to refine pricing methods for liability insurance contracts. In particular, the proposal is to model with a Generalised Pareto Distribution exactly the fraction  $r$  of largest losses which exhibits the Paretian tail. This can be achieved by adopting the threshold  $u$  (the minimum value of the loss over which a roughly linear behaviour of the mean-excess function can be observed) and the tail index  $\xi$  estimated by the EVT analysis. As a second step, for the left  $(1-r)$  part of the data (the lower losses), a classical model is considered. The idea is to combine the two models, requiring continuity and derivability in the transition point (the threshold).

The data involved in the analysis, showing more than one mode, suggested that a mixture of lognormal distributions should be used for modeling the lower losses, extending the approach of [3]. This is particularly suitable in the liability insurance context, where claims generated from a diversified portfolio (as, for example, personal injuries and property damages) can result in multiple density peaks.

To measure the goodness-of-fit, first the  $QQ$  plot has been considered, to check the conservative estimation for the highest quantiles. Then, the Mortara index  $A_1$ , the quadratic K. Pearson index  $A_2$  and a modified quadratic index  $A'_2$  have also been used.

Finally, we estimated the cost of claims on the ground of the fitted model, to show how remarkably useful are our results in applications.

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## Spatial modeling of annual minimum and maximum temperatures

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## Abstract

Bayesian hierarchical modeling is applied to the analysis of annual minimum and maximum temperatures. In both cases the generalized extreme value distribution is selected as the marginal distribution at each site due to its flexibility and theoretical basis. This distribution has three unknown parameters, a location parameter, a scale parameter and a shape parameter. The location and scale parameters are assumed to vary across sites while the shape parameter is assumed to be the same for all sites. Our model is very similar to the models presented in [1] and [2]. The location and the scale parameters are modeled as two independent Gaussian spatial processes which are governed by the Matérn correlation function. These spatial processes play a central role in spatial predictions at unobserved sites. The location parameter is also modeled with a common time effect which includes a linear trend and a linear site effect with altitude, distance to open sea, latitude and longitude as covariates. This model allows for spatial predictions for any set of unobserved sites, the scale of the grid can be as fine as possible as long as the covariates are observed at each of the unobserved sites.

Data on the annual minimum and maximum temperatures in Iceland from 1961 to 2009 at 72 sites are analyzed and used to predict the 2nd percentile of the minimum temperature for the 12-month period from July 2011 to June 2012 and the 98th percentile of maximum temperature for the year 2011 for a large set of unobserved sites across Iceland at which the covariates are available. The spatial predictions reveal that the 2nd percentile of minimum temperature for 2011 to 2012 reaches  $-35^\circ\text{C}$  in the central part of Iceland and ranges from  $-12$  to  $-22^\circ\text{C}$  around the coast of Iceland with the lowest coastal temperatures in the Northern part and the Northeastern part. The 98th percentile of maximum temperature in 2011 ranges from  $10$  to  $30^\circ\text{C}$  where the warmest areas are found in the Eastern part, the North-Eastern part and the Southern part of the country. The estimated increase in minimum and maximum temperatures over the years 1961 to 2009 is  $0.71$  and  $0.47^\circ\text{C}$  per decade, respectively, while the average annual temperature increased  $0.24^\circ\text{C}$  per decade over the same period.

## References

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# Extreme value models based on homogeneous and non-homogeneous Poisson process in climate change simulations

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## Abstract

The study compares statistical models for estimating high quantiles of daily temperatures based on the homogeneous and non-homogeneous Poisson process, and their applications to global climate model (GCM) simulations. Both types of the models make use of a non-stationary peaks-over-threshold method and the Generalized Pareto distribution (GPD) for modelling extremes, but they differ in how the dependence of the model parameters on time index is captured. The homogeneous Poisson process model assumes that the intensity of the process is constant and the threshold used to delimit extremes changes with time; the non-homogeneous Poisson process assumes that the intensity of the process depends on time while the threshold is kept constant [1]. The model for time-dependency of GPD parameters is selected according to the likelihood ratio test. Statistical arguments are provided to support the homogeneous Poisson process model, in which temporal dependence of the threshold is modelled in terms of regression quantiles [2]. Dependence of the results on the quantile chosen for the threshold (95-99%) is evaluated. The extreme value models are applied to estimate changes in high quantiles of daily temperatures (20-yr and 100-yr return values) in transient simulations of several GCMs for the 21st century.

## References

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# Location invariant Weiss-Hill estimator

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## Abstract

In this paper, a novel location invariant Weiss-Hill estimator of extreme value index  $\gamma \in \mathbf{R}$  is proposed. The new estimator is a combination of two estimators proposed by Weiss (1971) and Fraga Alves (2001a)[1], respectively. Its asymptotic behaviors such as weak and strong consistencies and asymptotic representations are obtained, including also a bias corrected location invariant Weiss-Hill estimator and the optimal choice of sample fraction asymptotic mean squared error (see also [2]). Simulation studies and comparison with other location invariant estimators given in [3],[4] are further considered.

## References

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# Conditional Analysis for Multivariate Extreme Financial Risks

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## Abstract

This poster discusses some recent theoretical development in the area of multivariate extreme value theory, based on the model proposed in Heffernan and Tawn (2004). The first half of the poster focuses on some of the unsolved issues in the original paper, namely the self-consistency issue and the residual distribution. The second half of the poster explores the an application of Heffernan-Tawn model in risk management and proposes a factor-based hierarchical multivariate extreme value model.

## Introduction

This section starts by reinforcing the importance of extreme risk management in the financial industry and moves onto a quick introduction to the background of multivariate extreme value theory. This is then followed by an overview of a conditional approach to analyze multivariate extreme value data, proposed in Heffernan and Tawn (2004), and the key concepts involved in the model.

## Theory

This section covers some of the recent research results developed from the original Heffernan and Tawn model, with regard to the the following two issues, which were previously unsolved

- Self-consistency requires the joint tail density being identical regardless of the conditioning margin

$$\mathbb{P}(Y_j = y_j | Y_i = y_i) f_{Y_i}(y_i) = \mathbb{P}(Y_i = y_i | Y_j = y_j) f_{Y_j}(y_j)$$

- The residuals of the standardized extreme observations converge to a non-degenerate yet unidentified distribution

So this section summarizes the potential problems that these issues are associated with and proposes a feasible resolution.

## Application

Motivated by some initial findings when applying the Heffernan and Tawn model to the real world financial data, this section introduces an intuitive multivariate extreme value model that accounts for the classes of assets in a typical investment portfolio. The main purpose of this section is to

- Provide some evidence to support a proposed factor-based hierarchical approach
- Describe the overall model formulation
- Illustrate its main application in cross-asset portfolio construction
- Discuss other potential applications in risk management

## Conclusion

This section summarizes the main findings to date and suggests areas of future work.

## References

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## Paleoclimate Extremes in Proxy Data

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## Abstract

There is much debate about the relationship between the changing climate and extreme climate events. Possible trends identified in temperature reconstructions based on proxy data lead to several questions about long-term climate behavior, and how to interpret this behavior given the patterns seen in proxy series. For example, using proxy data to address questions such as "Is there evidence that the extreme events of recent decades are more extreme than previous decades?" This leads to the question of whether the distributions of climate extremes are changing over time, which can provide important insights into how the climate system is changing. The methodology of extreme value theory has not been widely applied to this problem. This paper looks at what the statistics of extremes has to offer the field of paleoclimatology through modeling of the original proxy series, and seeks to address several of the emerging areas of research that intertwine extreme value analysis and paleoclimate reconstructions.

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## Strong invariance principles for tail quantile processes with applications to extreme value index estimation

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## Abstract

Many estimators of the extreme value index are functions of the  $k_n$  largest observations of the sample and therefore can be seen as a functional of the  $k_n$  upper tail quantile process. Under classical second order assumptions, this quantile process can be approximated, via a quantile transformation, by a non linear functional of the tail uniform empirical process (see [1]). Here, we prove a strong invariance principle for this non linear functional. In some ways, this result improves the approximation result obtained by [1] (Theorem 2.1) since it is convenient to prove strong limit theorems. In particular, we obtain a functional law of the iterated logarithm for the quantile process. As an application, we establish a compact law of the iterated logarithm for the classical Hill estimator [2].

## References

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- [2] B.M. Hill. A simple general approach to inference about the tail of a distribution, *Annals of Statistics*, **3**, 1163–1174, 1975.

## Statistical inference for the right endpoint of a light-tailed distribution

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## Abstract

In extreme value statistics we often encounter testing procedures for assessing the presence of the Gumbel domain, attached to the simple null hypothesis of the shape parameter being equal to zero. The problem of assessing for light-tailed distributions with finite or infinite right endpoint is seldom referred. We present two testing procedures which enable us to distinguish light-tailed distribution functions with finite right endpoint from those with infinite endpoint lying in the Gumbel domain. An estimator for the finite right endpoint is addressed with application to taxiway centerline deviation data.

# A non copula based model for extreme value dependence. Theory and applications.

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## Abstract

We present a new way of modeling dependent extreme value dependences. To date, the most common way of working with bivariate extremes is to separate and deal separately with the marginals and the dependence structure. See Coles et al in [1], and Jonathan Tawn et al in [2], for example. This approach proceeds by fitting the extreme marginals, by transforming the variables depending on the marginal distributions, and computing a couple of dependence measures with the transformed variables. Depending on the outcome, a type of copula would be chosen and then fitted.

In our model, both the marginal and the dependence structure are modeled simultaneously. This is done in such a way that the parameter modeling the tail dependence can be identified and estimated, i.e. the value of the tail dependence is a simple function of this parameter. Therefore, we have found a simple way to study whether two variables are tail dependent. We present simulation studies and a real data illustration involving financial data, which is very similar to the data used by [2], in order to compare the results of our model.

## References

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# Generalized real harmonisable multifractional stable process and it's path properties.

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## Abstract

The real harmonizable multifractional stable process (rhmsp for short) was introduced by Douzi and Shevchenko (2010), which has both properties of heavy tails and multifractionality, it can be regarded both as multifractional generalization of a harmonisable fractional stable process with Hurst parameter  $H$  and a stable generalization of harmonizable multifractional Brownian motion.

The rhmsp can be defined by replacing the Hurst parameter  $H$  by a Hölder function  $H(t)$ .

Our main interest in this talk is to introduce a new process how can be a generalization of rhmsp, which will be called a Generalized real harmonisable multifractional stable process. This process will also depend on a functional parameter  $H(t)$  that belongs to a set  $\mathcal{H}$ , but  $\mathcal{H}$  will be much more larger than the space of Hölder functions.

We also study its path properties: continuity, the localizability and the existence of local time.

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# Conditional Modelling of Extreme Values: Drug Induced Liver Injury

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## Abstract

Drug induced liver injury (DILI) is a major public health issue and of serious concern for pharmaceutical industry. Early detection of signs of a drug's potential for DILI is vital for pharmaceutical companies' evaluation of new drugs. Combination of extreme values of liver specific variables indicate DILI (Hy's Law). We estimate the probability of severe DILI using [1]

conditional dependence model which arises naturally in applications where a multidimensional random variable is extreme in at least one component. We extend the current model by including the assumption of stochastically ordered survival curves for different doses in a Phase 2 study.

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## On the distribution of the maximum of the Gaussian fields

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### Abstract

The problem of finding the distribution of the maximum of a random field has many applications in spatial statistics. In this poster, by using the Rice formulas developed by J.M. Azaïs and M. Wschebor [1] and extending the record method of C. Mercadier [5], we give some approximations of this distribution in the case of stationary Gaussian field for very general parameter sets.

In dimension 2, we provide an upper bound and a lower bound for the tail of the distribution at each level and prove that it is the exactly asymptotic result.

In dimension 3, from the result of the expectation of the absolute value of quadratic forms by W. Li and A. Wei [3], some similar results are obtained.

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## Estimation of extreme value index and two-step regression quantiles

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### Abstract

The contribution deals with estimators of extreme value index based on two-step regression quantiles in the linear regression model. Two-step regression quantiles can be seen as a possible generalization of the quantile idea and as an alternative to regression quantiles. We derive the approximation of the tail quantile function of errors. Following Drees (1998) we consider a class of smooth functionals of the tail quantile function as a tool for the construction of estimators in the linear regression context. Pickands, maximum likelihood and probability weighted moments estimators are illustrated on simulated data.

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## An empirical study of an adaptive resampling scheme for estimating the extremal index

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## Abstract

Extreme Value Theory deals with extreme or rare events, this is, events that have not been observed yet. It has been widely used in environmental applications, finance, hydrology, etc. There are several parameters that need to be estimated such as: tail index,  $\gamma$ , high quantiles, the return level and the *extremal index*,  $\theta$ . This parameter plays a special role in the description of the dependence between exceedances over a high threshold.

There exist several interpretations of the *extremal index* from which several estimators have been derived. One of those interpretations, due to [5], considers  $\theta$  as the reciprocal of the limiting mean cluster size. The identification of clusters of high level exceedances is then required.

The most basic form of cluster identification is to consider that a cluster occurs whenever there is an up-crossings of the high threshold  $u_n$ . This suggested the *up-crossing estimator*,  $\hat{\theta}_n^{UC}$ , ([6] and [1]), defined for a random sample  $(X_1, X_2, \dots, X_n)$  and a suitable threshold  $u_n$  as:

$$\hat{\theta}_n^{UC} := \frac{\sum_{i=1}^{n-1} I(X_i \leq u_n < X_{i+1})}{\sum_{i=1}^n I(X_i > u_n)}.$$

The estimation of the extremal index is then performed on the basis of the  $k$  order statistics in the sample or on the exceedances over a high level  $u_n$ . That estimator and other given in the literature, despite of having good asymptotic properties, present high variance for high levels and a high bias when the level decreases, showing a strong dependence on the high threshold  $u_n$ , for finite samples. Regarding the compromise between these two measures given by the mean squared error, MSE, a resampling scheme and an adaptive procedure is performed for estimating the optimal level  $u_n$  (that minimizes MSE) and then for obtaining an estimate of  $\theta$ . We are here mainly interested in the use of the bootstrap methodology for dependent data to estimate adaptively the optimal sample fraction.

For dependent data several bootstrap procedures have appeared in the literature, like the block bootstrap (non-overlapping blocks, “moving blocks”, circular blocks), sieve bootstrap, local bootstrap or jackknife-after-bootstrap.

The block bootstrap tries to mimic the behavior of an estimator by resampling blocks of consecutive observations; the blocking is used to preserve the original structure within a block. The performance of a block resampling method critically depends on the particular block length employed in finding the bootstrap estimator. [3] proposed a subsampling method to define a data-based version of *MSE* function which is minimized and rescaled to produce an estimator of the optimal block size. [4] proposed a plug-in rule as an alternative approach for empirical choice of the optimal block size. The key idea of the method is based on the bootstrap estimation of the variance and the bias of the block bootstrap estimator. The proposed rule is based on the Jackknife-After-Bootstrap (JAB) that yields a nonparametric estimator of the variance of a block bootstrap estimator, for the moving block bootstrap case, which is considered in this work.

Once obtained an estimate for the optimal block length, an adaptive procedure is carried out and the optimal sample fraction is obtained and finally  $\theta$  is estimated.

A simulation study as well as a real case study has been considered. The real data set consists of daily mean river levels from hydrometric station at Fraga, Portugal, during the years from 1946/47 to 1996/97. A subset of this data has already been studied to obtain the estimate of the extremal index, [2].

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## Accommodating Measurement Scale Uncertainty in Extreme Value Analysis of Northern North Sea Storm Severity

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## Abstract

Modelling storm severity is critical to the design and reliable operation of marine structures. Extreme hindcast storm peak significant wave heights ( $H_S$ ) for a neighbourhood of 50 locations in the northern North Sea are modelled, using the four

parameter Poisson point process model of [1], which incorporates measurement scale variability via a Box-Cox transformation. The model allows estimation of predictive distributions for both measurement scale and point process parameters within a Bayesian framework. The effect of measurement scale on return values of significant wave height is quantified by comparison with a three parameter Poisson point process model ignoring measurement scale uncertainty. It is found that, within the neighbourhood of locations examined, that the Box-Cox parameter varies from approximately 0.4 to 2, suggesting that appropriate measurement scales range from  $H_S^{\frac{2}{5}}$  to  $H_S^2$ . Moreover, return values corresponding to a return period of 3000 years can vary by as much as 15 to 20% when measurement scale uncertainty is accommodated.

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## Estimation of the truncation point of a truncated distribution

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### Abstract

The upper tail of the distribution of a random variable  $X$  can be truncated, what can be modelled by a truncated exponential distribution if the original uncut distribution belongs to the Gumbel or the Frechet domain of attraction. The variable  $X$  has to be transformed in last case by  $Y = \ln(X)$ . The consideration of the truncation point is not necessary in the most situations, but it influences considerably the distribution functions of wild fire sizes and earthquake magnitudes, what is motivation for the development and improvement of inference methods. There exist already estimation methods being described or introduced by Hannon and Dahiya [1]. Independent on this, estimation methods for the truncation point are discussed and developed in seismology (s.[2]). They base on different statistical approaches. The established inference methods are compared here with a new estimator [3] being presented in the poster. The new estimator has an excellent performance in a important range of parameters and has interesting properties from the users point of view. Beside this I show by numerical researches that the classical method of block maxima works only well for the estimation of the truncation point in special cases although the truncated exponential distribution belongs to the Weibull domain of attraction with extreme failure index  $\gamma = 1$  (s.[4]). This performance of the classical method of extreme value statistics explains the interpretation of "instability" by Pisarenko et al. [5]. Finally, the influence of the estimated truncation point to the inference of the scale parameter of the truncated exponential distribution is also researched.

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## New Estimating the conditional tail expectation in the case of heavy-tailed losses

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### Abstract

The conditional tail expectation (CTE) is a popular actuarial risk measure and a useful tool in financial risk assessment. Under the classical assumption that the second moment of the loss variable is finite, the asymptotic normality of the non-parametric CTE estimator has already been established in the literature. That result, however, is not applicable when the

loss variable follows any distribution with infinite second moment, which is a frequent situation in practice. With a help of extreme-value methodology, in this paper we offer a solution to the problem by suggesting a new CTE estimator, which is applicable when losses have finite means but infinite variances.

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## Flexible Extreme Value Mixture Modeling - Towards a Black Box?

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### Abstract

A plethora of recent articles have proposed various extreme value mixture models for threshold estimation, some of which also tackle the issue of quantifying the corresponding uncertainty. These mixture models typically treat the threshold as a parameter, so it can be objectively estimated using standard inference tools, avoiding the traditional graphical diagnostics which require expert (subjective) judgment. Some of these mixture models are easy to automate for application to multiple datasets, or in forecasting situations, for which in the past various ad-hoc adaptations had to be made in the past to overcome the threshold estimation problem.

This paper will outline one particularly flexible mixture model (MacDonald *et al*, 2011) which splices together the usual extreme value model for the upper tail behavior, with the threshold as a parameter, and the “bulk” of the distribution below the threshold captured by a non-parametric kernel density estimator. This representation avoids the need to specify a-priori a particular parametric model for the bulk distribution, and only really requires the trivial assumption of a smooth density which is realistic in most applications. The model flexibility also overcomes sensitivity to the specification of the bulk distribution (and in particular it’s lower tail). The existing mixture models in the literature which use parametric models for the bulk distribution can suffer from sensitivity to the specification of the lower tail, such that you can end up in the perverse situation where the fit for the lower tail can have strong influence on the inferences for the upper tail which is undesirable.

Inference for all the parameters, including threshold and kernel density bandwidth, is carried out in a Bayesian paradigm, potentially allowing sources of expert information to be included which can supplement the inherently sparse of extremal sample information.

A known problem with kernel density estimators is that they suffer from edge effects if the (lower) tail does not decay away to zero at the boundary. Various adaptations have been proposed in the nonparametric density estimation literature to overcome this problem. This paper explores one such approach which is flexible enough to cope with distributions which at the boundary have either: 1) a pole; 2) a shoulder; or 3) tail decaying to zero. An alternative approach of replacing the upper and lower tails by extreme models will also be shown to overcome these edge effects. The proposed boundary corrected extreme value mixture appears to provide a good step forward towards a black box solution for threshold estimation and uncertainty quantification for the well behaved population distributions that are typically observed in applications.

A version of the mixture model with extreme models for both tails is also shown to overcome two known problems with non-parametric density estimators: 1) tendency to oversmooth densities with heavy tails; and 2) sensitivity to outliers. In a quid pro quo, this model has led to a sharing of ideas and solutions between the extreme value and non-parametric density estimation literature.

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## Properties of the extremal correlation function and construction of spatial max-stable processes

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## Abstract

An adequate characterisation of the spatial dependence of a stationary max-stable random field  $(X_t)_{t \in T}$  is a non-trivial issue, specifically in view of the non-existence of covariances for certain marginal distributions. Being a bivariate positive definite characteristic that can be inferred from data the extremal correlation function

$$\chi(t) := \lim_{x \rightarrow \infty} \mathbb{P}(X_t \geq x | X_0 \geq x)$$

has been proposed as an analogue of the covariance function [1].

We investigate the set of all valid extremal correlation functions  $\chi(t)$  on general spaces  $T$ , including  $\mathbb{Z}^d$  or  $\mathbb{R}^d$ , and relate it to other collections of positive definite functions with values in  $[0, 1]$ , namely set covariance functions, uncentered covariance functions of binary fields, and pointwise limits of extremal correlation functions.

In particular, we raise the following question: Does the set of extremal correlation functions coincide with the set of functions  $f(t) = \lim_{x \rightarrow \infty} \mathbb{P}(X_t \geq x | X_0 \geq x)$  where  $X$  is a (not necessarily max-stable) process such that the respective limit exists? For processes on  $T = \mathbb{Z}$  this is true, for example. Moreover, in this case any extremal correlation function  $\chi$  with bounded support can be represented as a set covariance function [2]. This allows the construction of a max-stable process realising  $\chi(t)$ .

As a byproduct, we generalise a distribution model from [1] and obtain an explicit model for any valid collection of extremal coefficients

$$\theta(A) := \frac{-\log \mathbb{P}(\max_{t \in A} X_t \leq x)}{-\log \mathbb{P}(X_t \leq x)}.$$

Thus, we also establish an intriguing connection to the capacity functional of a binary field (compare [3]).

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## Stable estimations for extreme wind speeds

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## Abstract

The Generalized Pareto distribution (GPD)

$$G(y) = 1 - \left(1 + \gamma \frac{y}{\sigma}\right)^{-1/\gamma}, \quad (1)$$

is frequently applied for the statistical analysis of extreme wind speeds. Here,  $y$  is the excess over a sufficiently high threshold,  $\gamma$  the extreme-value index or the shape parameter, and  $\sigma$  is the scale parameter. A central topic in extreme-value theory is the adaptive estimation of  $\gamma$ . A characteristic feature of wind speed measurements is the absence of decimal units. Unfortunately, such a loss accuracy may seriously affect the parameter estimation of GPD. In this context, several authors have demonstrated a high sensitivity of  $\gamma$  against the threshold when analysing extreme wind speeds, see for example [2]. This undesirable effect introduces the difficulty to provide reliable quantile estimates. In [3] we aim to bring this problem to meteorologists and proposes the estimator of Beirlant *et al.* [1] (the Zipf-estimator) for  $\gamma$ . The method is based on regression in the so-called generalized quantile plots. We summarize our main conclusions of the comparative tests with the Zipf-estimator and classical estimators (probability weighted method, maximum likelihood, ...). Firstly, we have shown that the estimation of  $\gamma$  becomes stable as the threshold moves when using the Zipf-estimator. This could allow a more objective prior identification of the sign and range of  $\gamma$ . Taking into account that the observations were rounded would change the tail estimation rather little for the Zipf-estimator. In contrast, the rounding increased substantially the errors in the classical parameter estimation. Secondly, extensive Monte-Carlo simulations indicate that the Zipf-estimator is much more accurate and reliable than classical methods. These simulations were designed in such a way that they share the characteristics of Belgian extreme wind speeds as close as possible. Thirdly, convergence of  $\gamma$  provided by the Zipf-estimator is generally achieved for lower threshold values compared to classical methods, so that more data can be included in the extreme-value analysis. Finally, the new methodology is applied to get improved prediction of extreme wind gusts in Belgium. A considerably shortening of the confidence intervals for the parameters and quantiles is found when the Zipf-estimator is used. Furthermore, the spatial differences of GPD's parameter are greatly reduced.

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## Rates of convergence of extremes for mixed exponential distributions

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### Abstract

One interesting topic in extreme value theory is to consider the uniform convergence rates of distributions of extremes to its ultimate extreme value distribution. de Haan and Resnick [2], under the second Von-Mises conditions, considered the convergence rate in the uniform metric and the total variance metric. For the uniform convergence rates of the extremes of some special distributions, see Hall [3] for normal distribution and Hall and Wellner [4] for exponential distribution. Recently Peng et al. [5] extended Hall's work to the general error distribution, and Lin et al. [1] considered the uniform convergence rates of extremes of short tailed symmetric distribution.

Let  $F$  denote the finite mixed exponential distribution defined by

$$F(x) = p_1F_1(x) + p_2F_2(x) + \cdots + p_rF_r(x),$$

where weights  $0 < p_i < 1$ ,  $\sum_{i \leq r} p_i = 1$  and  $F_i$  follows exponential distribution with parameter  $\lambda_i$  for  $1 \leq i \leq r$ . In this short note, we consider the limiting distributions of the extremes of finite mixed exponential distribution and the associated uniform converge rates. The practical values of the results are illustrated by a numerical study.

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